Simple Harmonic Motion

Question1

A particle is executing simple harmonic motion. If the force acting on the particle at a position is 86.6% of the maximum force on it, then the ratio of its velocity at that point and its maximum velocity is

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Options:

A.

$1 : \sqrt{3}$

B.

$1 : 2$

C.

$\sqrt{3} : 2$

D.

$1 : 3$

Answer: B

Solution:

The force in SHM is $F = -kx$

The maximum force is $F_{\text{max}} = -kA$

Given, $F = 0.866F_{\text{max}}$

$$So, \ -kx = 0.866(-kA)$$
$$x = 0.866 \text{ A}$$

The velocity in SHM is $v = \omega\sqrt{(A^2 - x^2)}$

$$v = \omega\sqrt{A^2 - (0.866A)^2}$$
$$v = \omega\sqrt{A^2 - 0.75A^2} = \omega\sqrt{0.25A^2}$$
$$v = \omega(0.5A)$$

The maximum velocity in SHM occurs at $x = 0$, So $v_{\max} = \omega A$

The ratio is $\frac{v}{v_{\max}} = \frac{0.5\omega A}{\omega A}$

$\frac{v}{v_{\max}} = 0.5 = \frac{1}{2}$

The ratio of its velocity at that point and its maximum velocity is $\frac{1}{2}$.

Question2

The amplitude of a particle executing simple harmonic motion is 6 cm . The distance of the point from the mean position at which the ratio of the potential and kinetic energies of the particle becomes $4:5$ is

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Options:

A.

6 cm

B.

4 cm

C.

3 cm

D.

2 cm

Answer: B

Solution:

Given:

$\frac{U}{K} = \frac{4}{5}$

The amplitude $A = 6$ cm

Step 1: Write the energy formulas

The total energy is $E = \frac{1}{2}kA^2$

The potential energy is $U = \frac{1}{2}kx^2$

The kinetic energy is $K = \frac{1}{2}k(A^2 - x^2)$

Step 2: Set up the given ratio

We want the distance x from the mean position so that $\frac{U}{K} = \frac{4}{5}$:

$\frac{\frac{1}{2}kx^2}{\frac{1}{2}k(A^2 - x^2)} = \frac{4}{5}$

Step 3: Simplify the equation

The $\frac{1}{2}k$ cancels out: $\frac{x^2}{A^2 - x^2} = \frac{4}{5}$

Step 4: Solve for x^2

Multiply both sides by $(A^2 - x^2)$: $5x^2 = 4(A^2 - x^2)$

Expand: $5x^2 = 4A^2 - 4x^2$

Bring $4x^2$ to the left: $5x^2 + 4x^2 = 4A^2$

Combine like terms: $9x^2 = 4A^2$

Step 5: Find x

Divide both sides by 9: $x^2 = \frac{4}{9}A^2$

Take the square root: $x = \frac{2}{3}A$

Since $A = 6$ cm:

$x = \frac{2}{3} \times 6 = 4$ cm

The distance from the mean position is 4 cm.

Question3

In a simple pendulum experiment for the determination of acceleration due to gravity, the error in the measurement of the length of the pendulum is 1% and the error in the measurement of the time period is 2%. The error in the estimation of acceleration due to gravity is

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Options:

A. 1%

B. 3%

C. 4%

D. 5%

Answer: D

Solution:

In a simple pendulum experiment, time period T of a pendulum is given by,

$T = 2\pi\sqrt{\frac{L}{g}}$

where, $T =$ time period of pendulum, $L =$ length of pendulum

$g =$ acceleration due to gravity rearrange this formula to solve for g,

$T^2 = 4\pi^2 \cdot \frac{L}{g}$

$g = 4\pi^2 \cdot \frac{L}{T^2}$

To find the percentage error in g, differentiate the above equation and we get

$\frac{\Delta g}{g} \times 100 = \frac{\Delta L}{L} \times 100 + 2\frac{\Delta T}{T} \times 100$

Given, $\frac{\Delta L}{L} \times 100 = 1\%$ and $\frac{\Delta T}{T} \times 100 = 2\%$

So, $\frac{\Delta g}{g} \times 100 = 1\% + 2 \times 2\%$

$\frac{\Delta g}{g} \times 100 = (1 + 4)\%$

$\frac{\Delta g}{g} \times 100 = 5\%$

Question4

A massless spring of length l and spring constant k oscillates with a time period T when loaded with a mass m. The spring is now cut into three equal parts and are connected in parallel. The frequency of oscillation of the combination when it is loaded with $_3$ mass 4 m is

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Options:

A. $\frac{2}{T}$

B. $\frac{2}{3\pi}$

C. $\frac{3}{T}$

D. $\frac{3}{2T}$

Answer: D

Solution:

For a mass m oscillating with a spring of constant k and length l, the time period T is given by,

$T=2\pi\sqrt{\frac{m}{k}}$

When spring of length l is cut into three equal parts, each part has a length $\frac{l}{3}$. The new spring constant, k' is

$k'=3k$

Equivalent spring constant for parallel combination

$$k_{\text{eq}}=k'+k'+k'=3k'$$

$$=3\times3k=9k$$

Now, we load parallel combination with a mass of 4 m . The new time period using equivalent spring constant, is

$T'=2\pi\sqrt{\frac{4m}{k_{\text{eq}}}}=2\pi\sqrt{\frac{4m}{9k}}=\frac{4\pi}{3}\sqrt{\frac{m}{k}}$

From Eq. (i),

$T'=\frac{2}{3}T$

The frequency of oscillation f' is the reciprocal of the time period T'. Therefore,

$f' = \frac{1}{T'}$

$f' = \frac{1}{\frac{2}{3}T}$

$f' = \frac{3}{2T}$

Question5

If a body dropped freely from a height of 20 m reaches the surface of a planet with a velocity of $31.4 \mathrm{~ms}^{-1}$. then the length of a simple pendulum that ticks seconds on the planet is

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Options:

A. 1 m

B. 0.625 m

C. 2.5 m

D. 2 m

Answer: C

Solution:

Given, $h=20 \mathrm{~m}$

$u=0, v=31.4 \mathrm{~m} / \mathrm{s}$

Time period of simple pendulum $=2 \mathrm{~s}$

Time period of simple pendulum is gives as

$T=2 \pi \sqrt{\frac{l}{a}}$

($l=$ length, $a=$ acceleration)

Using equation of motion to find a

$v^2 = u^2 + 2as$

$(31.4)^2 = 0 + 2 \times 20 \times a$

$(3.14)^2 \times 100 = 40a$

$a = (3.14)^2 \times \frac{100}{40}$

$= (3.14)^2 \times 2.5$

Substitute the value of a and T in Eq. (i)

$2 = 2\pi\sqrt{\frac{1}{(3.14)^2 \times 2.5}}$

$1 = \frac{3.14}{3.14}\sqrt{\frac{l}{2.5}}$

$1 = \sqrt{\frac{l}{2.5}}$

Squaring both sides

$l = 2.5 \text{ cm}$

Question6

A particle of mass 4 mg is executing simple harmonic motion along X-axis with an angular frequency of 40rads^{-1}. If the potential energy of the particle is $V(x) = a + bx^2$, where $V(x)$ is in joule and x is in metre, then the value of b is

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Options:

A. $800 \times 10^{-6} \text{Jm}^{-2}$

B. $1600 \times 10^{-6} \text{Jm}^{-2}$

C. $3200 \times 10^{-6} \text{Jm}^{-2}$

D. $6400 \times 1^{-6} \text{Jm}^{-2}$

Answer: C

Solution:

Given, mass of particle,

$m = 4\text{mg} = 4 \times 10^{-6}$ kg

angular frequency, $\omega = 40\text{rads}^{-1}$

Potential energy of the particle,

$V = a + bx^2$

We know that $F = -\frac{dV}{dx} = -2bx$

also

$F = -kx = -2bx$

$\Rightarrow \quad k = 2b$

Also $\frac{k}{m} = \omega^2$

$\Rightarrow \quad k = \omega^2 m = 2b$

$$\Rightarrow \quad b = \frac{\omega^2 m}{2}$$

$$= \frac{(40)^2 \times 4 \times 10^{-6}}{2}$$

$$= 3200 \times 10^{-6}\ \text{J/m}^2$$

Question7

In a time t amplitude of vibrations of a damped oscillator becomes half of its initial value, then the mechanical energy of the oscillator decreases by

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Options:

A. 40%

B. 20%

C. 75%

D. 50%

Answer: C

Solution:

The mechanical energy (E) of a damped oscillator is proportional to the square of its amplitude (A). When the amplitude becomes half of its initial valuc, the mechanical energy decrease to a quarter of its initial value. This is

because the mechanical energy is given by

$E \propto A^2$

So, if the initial energy is E_0 and the initial amplitude is A_0. When the amplitude becomes $\frac{A_0}{2}$.

$$E = E_0\left(\frac{\frac{A_0}{2}}{A_0}\right)^2 = E_0\left(\frac{1}{2}\right)^2 = \frac{E_0}{4}$$

$\therefore$ Decrease in mechanical energy of oscillator

$$= E_0 - \frac{E_0}{4} = \frac{3E_0}{4}$$

$\therefore$ % decrease in mechanical energy

$$= \frac{\frac{3E_0}{4}}{E_0} \times 100 = 75\%$$

Question8

A clock is designed based on the oscillation of a spring-block system suspended vertically in the absence of air-resistance. Assume it shows the correct time when a spring of stiffness k and block is mass m are used. If the block is replaced by another block of mass $4m$, choose the correct option

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Options:

A. The clock runs slow by 0.5 s for every second

B. The clock runs fast by 0.5 s for every one second

C. The clock runs fast by 1 s for every one second

D. The clock runs slow by 1 s for every one second

Answer: A

Solution:

Initial Setup:

Spring constant: k

Mass: m

Initial time period: 1 s

New Setup:

Replaced mass: $4m$

For a spring-block system in simple harmonic motion (SHM), the time period t is given by:

$t = 2\pi\sqrt{\frac{m}{k}}$

In the initial setup, we have:

$1 = 2\pi\sqrt{\frac{m}{k}}$

Rearranging gives:

$\frac{1}{4\pi^2} = \frac{m}{k} \quad \Rightarrow \quad k = \frac{m}{4\pi^2}$

Now, substituting this new mass into the time period formula:

$t_2 = 2\pi\sqrt{\frac{4m}{k}}$

Replacing k with the expression found:

$t_2 = 2\pi\sqrt{\frac{4m}{\frac{m}{4\pi^2}}} = 2\pi\sqrt{\frac{4\times4\pi^2}{1}} = 2\pi\sqrt{16\pi^2}$

This simplifies to:

$t_2 = 0.5$ s

Thus, the clock now runs slow by 0.5 seconds for every second that passes.

Question9

For a particle executing simple harmonic motion, the kinetic energy of the particle at a distance of 4 cm from the mean position is $1/3$ rd of the maximum kinetic energy. The amplitude of the motion is

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Options:

A. $2\sqrt{6}$ cm

B. $\frac{2}{\sqrt{6}}$ cm

C. $\sqrt{2}$ cm

D. $\frac{6}{\sqrt{2}}$ cm

Answer: A

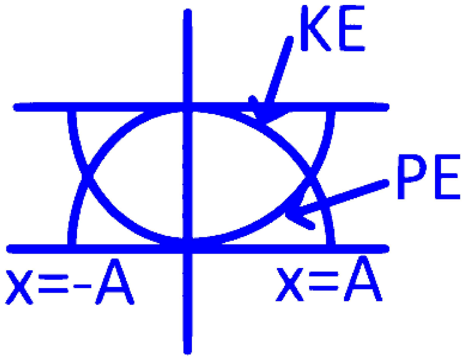
Solution:

Given,

Particle is executing SHM at $x_1 = 40\text{ml}$ from mean position.

$\text{KE} = \frac{1}{3}\text{KE}_{\text{max}}$

max potential energy of SHM$



$$\text{KE}_{\text{max}} = \text{TE} = \frac{1}{2}KA^2$$
$$\text{KE} = \frac{1}{3}(\text{KE}_{\text{max}}) = \frac{1}{3} \times \frac{1}{2}KA^2$$
$$= \frac{1}{3} \times \frac{1}{2}KA^2$$
$$\frac{1}{2}K\left(A^2 - x^2\right) = \frac{1}{6}KA^2$$
$$A^2 - x^2 = \frac{1}{3}A^2$$
$$Atx = 4 \text{ cm}$$
$$A^2 - 16 = \frac{1}{3}A^2$$
$$\therefore A = 2\sqrt{6} \text{ cm}$$
